Exercise 28

The population of a certain species in a limited environment with initial population 100 and carrying capacity 1000 is

$$P(t) = \frac{100\,000}{100 + 900e^{-t}}$$

where t is measured in years.

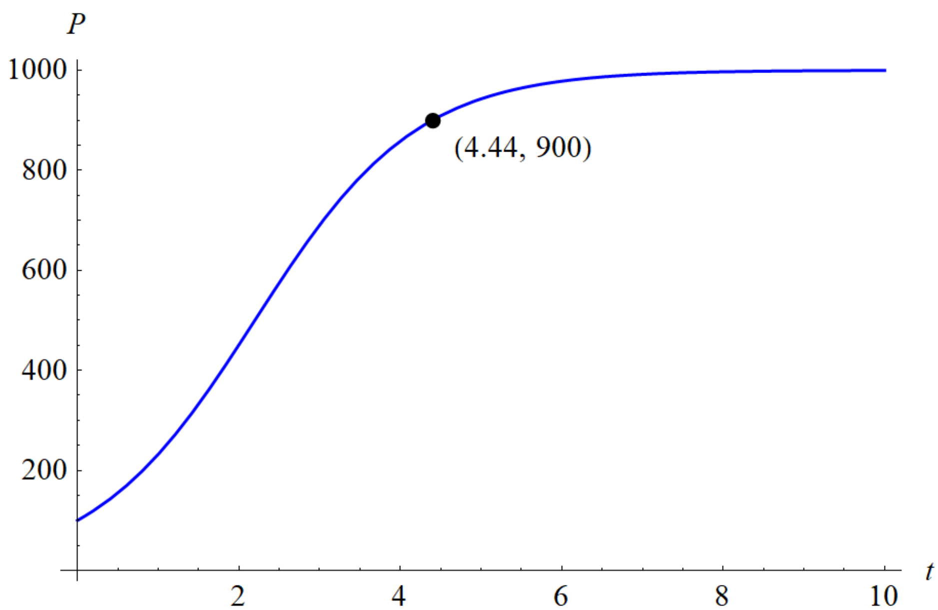
(a) Graph this function and estimate how long it takes for the population to reach 900.

(b) Find the inverse of this function and explain its meaning.

(c) Use the inverse function to find the time required for the population to reach 900. Compare with the result of part (a).

Solution

Graph the given function and label the point where the population reaches 900.



It takes about four and a half years. Solve the population function for t.

$$P(t) = \frac{100\,000}{100 + 900e^{-t}} = \frac{1000}{1 + 9e^{-t}}$$

$$1 + 9e^{-t} = \frac{1000}{P(t)}$$

$$9e^{-t} = \frac{1000}{P(t)} - 1$$

$$e^{-t} = \frac{1000}{9P(t)} - \frac{1}{9}$$

Take the natural logarithm of both sides.

$$\ln e^{-t} = \ln\left[\frac{1000}{9P(t)} - \frac{1}{9}\right]$$

Bring the exponent down in front.

$$-t\ln e = \ln\left[\frac{1000}{9P(t)} - \frac{1}{9}\right]$$

Multiply both sides by -1 and use the fact that $\ln e = 1$.

$$t = -\ln\left[\frac{1000}{9P(t)} - \frac{1}{9}\right]$$

This is the inverse function; it gives the time it takes to reach a certain population. Plug in $P(t) = 900$ and evaluate the formula.

$$t = -\ln\left[\frac{1000}{9(900)} - \frac{1}{9}\right] = \ln 81 \approx 4.394$$